Bachelor of Science (B.Sc.) Semester–III (C.B.S) Examination

STATISTICS (Statistical Methods)

Paper — I

Time : Three Hours] [Maximum Marks : 50

Note :— All questions are compulsory and carry equal marks.

1. (A) If the joint probability density of X and Y is given by :

$$f(x, y) = \begin{cases} \frac{2}{3}(x + 2y), & 0 < x < 1, \ 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find :

(i) Marginal density of Y

(ii) Conditional density of $X / Y = \frac{1}{2}$

(iii) Conditional mean of $X / Y = \frac{1}{2}$

(iv) Conditional variance of $X / Y = \frac{1}{2}$ 10

OR

(E) If X is the number of heads and Y, the number of heads minus the number of tails obtained in three tosses of a balanced coin, construct a table showing the values of the joint probability distribution of X and Y. Also find the marginal distributions of X and Y. 10

2. (A) State the p.d.f. of binomial distribution. Find its m.g.f. and hence state m.g.f. of marginal distributions of random variables. Also state means and variances of these distributions.The complexity of arrivals

and departures into an airport are such that computer simulation is often used to model the ideal conditions. For a certain airport containing three runways it is known that in the ideal setting the respective probabilities for runways 1, 2 and 3 are $\frac{2}{9}$, $\frac{1}{6}$ and $\frac{11}{18}$, that are accessed by a random commercial jet. What is the probability that 6 randomly arriving airplanes are distributed in the following fashion ?

Runway 1 : 2 airplanes

Runway 2 : 1 airplanes

Runway 3 : 3 airplanes. 10

OR

(E) State the p.d.f. of Bivariate normal distribution. Find marginal density of Y and conditional density of X given Y = y. 10

3. (A) Write the steps for drawing a random sample of size n from Normal population with mean μ and variance σ^2.

(B) Let X have the p.d.f. $f(x) = 3(1 - x)^2$, $0 < x < 1$.

Find the p.d.f. of $Y = (1 - X)^3$.

(C) If X follows poisson distribution with parameter 4, find the p.m.f. of $Y = \sqrt{X}$.

(D) If the joint distribution of X and Y is $f(x, y) = \frac{(x-y)^2}{7}$, x = 1, 2, y = 1, 2, 3,

find (i) the joint distribution of U = X + Y and V = X – Y.

(ii) the marginal distribution of U. 2½×4=10

OR

(E) If X and Y are two chi-square variates with n_1 and n_2 d.f. respectively then show that $\frac{X}{Y}$ is a $\beta_2\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$ variate.

(F) Let X_1 and X_2 be two independent standard normal variables. Find the joint p.d.f. of $Y_1 = X_1 + X_2$ and $Y_2 = X_2 - X_1$. 5+5

4. (A) Define F-statistic. Derive its p.d.f. Find mean and mode of this distribution. Hence comment on the skewness. 10

OR

(E) Define Fisher's t. State its p.d.f. Define student's t. Show that it may be regarded as a particular case of Fisher's t.

(F) Find m.g.f. of chi square distribution. Hence find mean and variance of it. 5+5

5. Solve any **ten** of the following :

(A) If $X \sim F(n_1, n_2)$, state the distribution of $\frac{1}{X}$.

(B) Define a Chi-square Statistic.

(C) If $X \sim t_n$ then state the distribution of X^2.

(D) If X and Y have Bivariate normal distribution with parameters

$$\mu_x = 70,\ \mu_y = 80,\ \sigma_y^2 = 169,\ \rho = \frac{5}{13},\ \sigma_x^2 = 100$$

Find V(Y/X = 90).

(E) If a pair of r. vs X and Y follow Bivariate normal distribution, state its m.g.f.

(F) In Bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2$ and ρ, if $\rho = 0$, state marginal distributions of random variables.

(G) Define joint p.m.f. of random variables X and Y.

(H) Define correlation coefficient between two random variables.

(I) In usual notation, if random variables X and Y are independent, show that

$M_{XY}(t_1, t_2) = M_X(t_1, 0) \cdot M_Y(t_2, 0)$.

(J) Define a Random sample.

(K) Define Sampling distribution.

(L) If F(x) is the value of the distribution function of the continuous r. v. X at x, find the probability distribution of Y = F(x). 1×10=10